ACCELERATION PHENOMENA OF HARDWARE COUPLED OSCILLATORS FOR CARDIAC PURKINJE FIBER CELLS

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ABSTRACT

Excitable or oscillatory electrical phenomena, shown by neurons and many other cells, are modeled by using nonlinear differential equations [1]. These models are often used as the components in the networks to analyze the electrical waveform of the action potential, because its dynamics is regarded as main information carrier in the biological systems. Meunier[2], for instance, showed surprising phenomena on the acceleration of the oscillation in the electrical coupling of the modified FitzHugh-Nagumo (FHN) [1] oscillators. When two nonlinear oscillators that have slightly different frequencies are coupled diffusively, the frequencies of the synchronized waveforms get larger. The period of waveforms for such acceleration phenomena were mathematically analyzed by using a piece-wise linear FHN equation.

However, it is still difficult to understand the physical and physiological meanings of the acceleration from the simple mathematical models. Generally, the physical quantities such as voltage, current and power disappear from the abstract mathematical models, because the variables in the equation are dimensionless. In order to understand the dynamics intuitively, a model described by the electronic circuits is very useful. To this end, many types of hardware models have proposed [3],[4]. In this paper, we proposed the diffusively or electrically coupled hardware models for a cardiac Purkinje fiber cell to clarify the acceleration of the oscillation.

Maeda and Makino [4] obtained the burst generating hardware model by means of modifying the Hoshimiya[3]'s excitable membrane model. This modification method is based on general results of the diversity and common features in the nonlinear Hodgkin-Huxley type systems [5], and leads also to the circuit model (Fig. 1) for the Purkinje fiber cell that generates the plateau duration after the sharp upstroke in the temporal waveforms. In Fig. 1, four dotted arrows represent the voltage-dependent inward sodium current I_{Na1}, the small sodium current I_{Na2} with the fixed inward bias, the instantaneous outward potassium current I_{K1} and the delayed rectifier potassium current I_{K2}, respectively. The plateau duration is maintained by the anti-synergistic currents, I_{Na1} and I_{K1}. The current I_{Na2} is called the pacemaker current, and, as such, initiates another action potential after one period of the waveform. The remaining current I_{K2} is responsible for terminating the plateau duration. The sodium equilibrium potential V_{Na} and the potassium equilibrium potential V_K were fixed as 5 and 0 volts, respectively. So, the waveforms are observed between 0 and 5 volts (or between V_K and V_{Na}). Actually, the voltage source V_K is omitted from the circuit because $V_K = 0$.

We regulated the parameter set (R_i, C_s), so that the electrically coupled system of the hardware models could show the acceleration of the oscillation, as shown in Fig. 2(a1), (a2), (b1) and (b2). In

this case one "inside", depicted by the upper side in Fig. 1, was connected to the "inside" of the other circuit. Physiologically, the "inside" means the inside of the membrane.

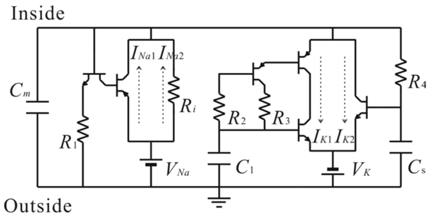

Fig. 1: Hardware model for the cardiac Purkinje fiber cell; V_{Na} =5, V_K =0, $C_m = C_1$ =10u, R_1 =200k, $R_2 = R_4$ =100k, R_3 =10k. Resistor R_i and capacitor C_s are variable parameters. Transistors: c2458Y9D(npn), c1048Y6E(pnp).

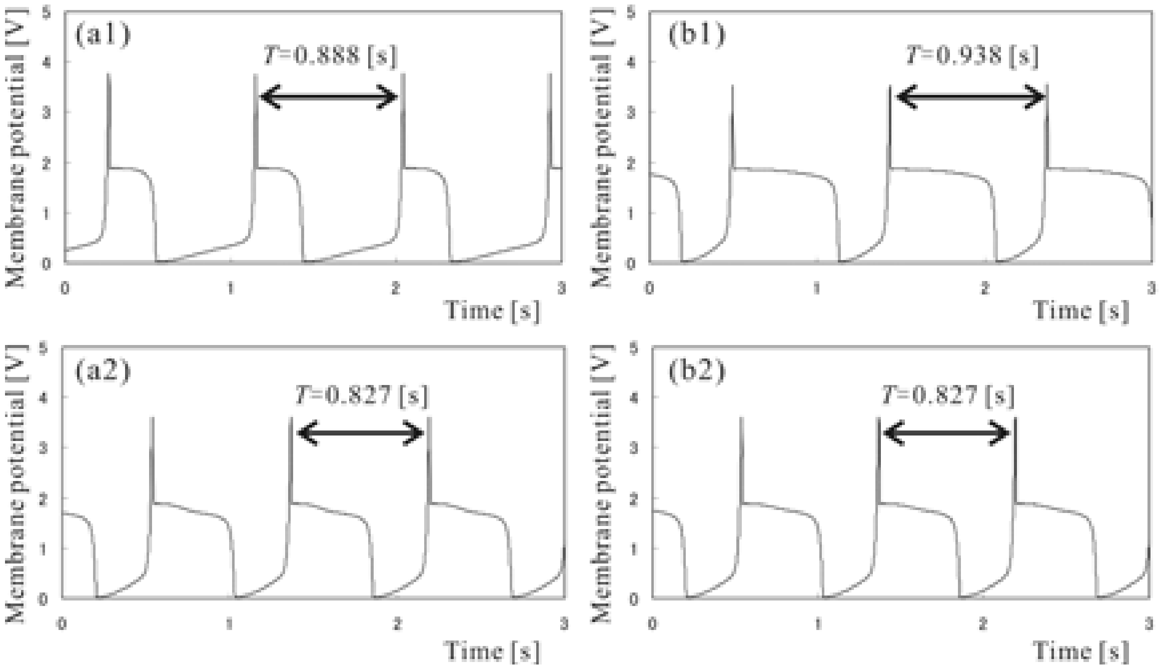

Fig. 2: Temporal waveforms before and after coupling. (a) Model-I: R_i = 714k, C_s = 15u, (b) Model-II: R_i = 43.9k, C_s = 105u. (a1)(b1) before coupling, (a2)(b2) after coupling. Abscissa is a time in seconds and ordinate the membrane potential in volts. (c) Electrical coupling of the hardware models. $R_D = 100$.

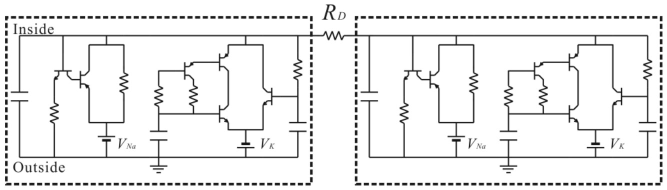

Fig. 3: Electrically coupled system of the hardware models

Figure 3 exhibited the circuit configuration of the coupled system. The coupling resistance R_D was located between the insides (Fig. 3). This sort of electrical coupling physiologically corresponds to the *gap junction* in the cardiac system. The data was forwarded to the personal computer through an A/D converter (EC-2360, sampling frequency 1kHz), to evaluate the period T , the plateau duration T_a , and the ratio of the plateau duration of the period, so-called, the activity A :

$$A = T_a / T \cdot \quad (1)$$

When the Model-I was connected to the Model-II , the frequency f of the coupled system was larger than those of the Model-I and -II (referred to the middle column in Table 1). Why the oscillation was accelerated? We empirically found the fact that the activity of the coupled system would be in the middle of the activity inherent in each, even though the oscillation should be accelerated (referred to the right column in Table 1). The activity must be correlated to the electric power, because it is necessary for keeping up the potential plateau, or the high activity, to flow the current from the voltage source V_{Na}.

Table 1: periods, frequencies and activities

	T	$f=1/T$	A
Model-I	0.888	1.126	0.328
Model-II	0.938	1.066	0.678
Coupled system	0.827	1.209	0.601

We investigated the relations among the acceleration, the activity, and the electric power, by using the simulation software, B^2-*Spice A/D 2000* (Beige Bag Software, Inc.). Because the V_{Na} is the only source of energy into the circuit system, we introduced the electric power as follows:

$$P = \frac{1}{T}\int_0^T V_{Na} \cdot I_{Na}(t)\,dt\,, \tag{2}$$

where $I_{Na}(t) = I_{Na1}(t) + I_{Na2}(t)$.

Figure 4(a) exhibited the shrinkage of the synchronized period against the difference between the activities (*DBA*) of two hardware models, the Model-A and the Model-B. Both intrinsic periods before synchronization were fixed as one second, and the activity of the Model-B was also fixed as 0.615 ($T_a = 0.615$ seconds). The *DBA* was set by means of manipulating the activity of the Model-A. The graph in Fig. 4(a) was not monotonous and there was minimum value around *DBA* = 0.4. When the *DBA* was about 0.4, the minimum synchronized period was about 0.7 seconds and in this case the oscillation was most accelerative.

Figure 4(b) exhibited the shrinkage of the electric power (*SEP*) of the coupled system after synchronization, against the *DBA*. We evaluated the *SEP*, using (2), as follows:

$$SEP = (P_A^{(sync)} + P_B^{(sync)}) - (P_A + P_B). \tag{3}$$

This graph also showed the minimum point where the *DBA* was approximately equal to 0.4. This means that the acceleration might be correlative to the electric power. In fact, the correlation coefficient between the synchronized period and the electric power was evaluated 0.62. If the information carrier of the biological systems meant the action potentials, the biological systems would propagate swiftly the information through the mutual connection of the cells.

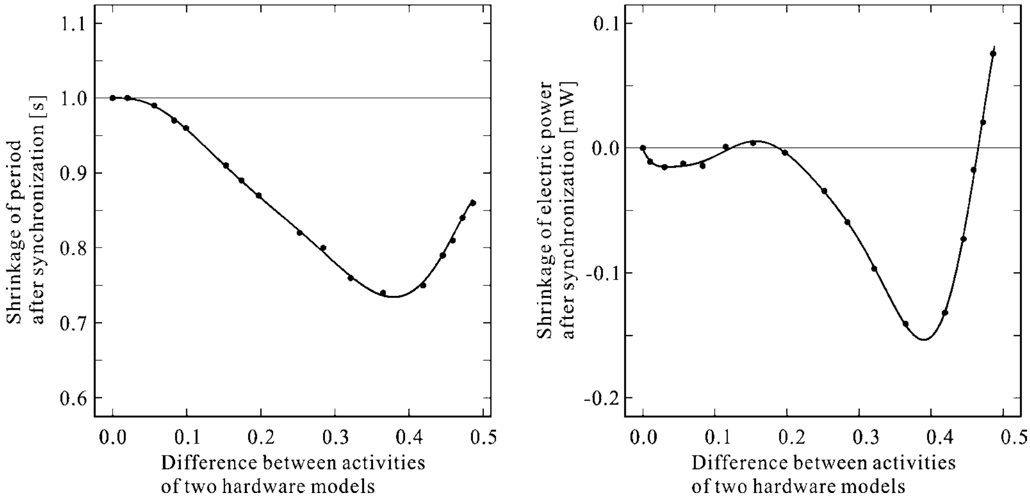

Fig. 4: (a) Shrinkage of the period after synchronization, (b) Shrinkage of the electric power (*SEP*) after synchronization. Abscissas are the difference between the activities (*DBA*) of two hardware models.

We have investigated the acceleration phenomena on the hardware systems mainly to understand their aspect of the electric power. As the result, we suggested that there should be a positive correlation between the shrinkage of the period, that is, the acceleration of the oscillation, and the shrinkage of the electric power in the coupled system. Our research in this paper was summarized as follows:

1) We constructed the hardware model for a cardiac Purkinje fiber cell, by means of modifying the burst generating hardware neuron model.
2) We found that the frequency of each hardware model could be high, in other words, the oscillation could be accelerated in the electrically coupled system. In this case the hardware models must take different values of the activities.
3) When the acceleration phenomena were observed, the electric power of the system could be decreased.

Keywords: Plateau potential, Diffusive coupling, Nonlinear circuit oscillator.

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