A SIMULATION APPROACH TO NEURONAL ACTIVITY

Antonio Di Crescenzo
Dipartimento di Matematica e Informatica, Università di Salerno
Via S. Allende, I-84081 Baronissi (SA), Italy
adicrescenzo@unisa.it

Barbara Martinucci, Enrica Pirozzi
Dipartimento di Matematica e Applicazioni, Università di Napoli Federico II
Via Cintia, I-80126 Napoli, Italy
barbara.martinucci@dma.unina.it, enrica.pirozzi@unina.it

ABSTRACT

The well-known Stein's neuronal model is based on the assumption that changes in the membrane potential between two consecutive neuronal firings are described by the one-dimensional stochastic process $\{X(t);\ t \geq 0\}$, that obeys the stochastic differential equation (Stein [3])

$$\mathrm{d}X(t) = -\frac{1}{\tau}\, X(t)\,\mathrm{d}t + a\,\mathrm{d}N^{+}(t) + i\,\mathrm{d}N^{-}(t), \qquad X(0) = x_0,$$

where $\tau > 0$, $i < 0 < a$ are constants, $\{N^{+}(t);\ t \geq 0\}$ and $\{N^{-}(t);\ t \geq 0\}$ are independent homogeneous Poisson processes. Starting from the Stein's model we consider a simple stochastic model to describe the time-evolution of the neuronal membrane potentials for the units of a neural network.

In this model the essential assumptions concerning each unit of the network are the following:
– in absence of stimuli the membrane potential exponentially decays to the resting level;
– the effects of excitatory and inhibitory stimuli, that occur according to independent Poisson processes, are jumps with positive and negative amplitude, respectively, for the membrane potential;
– when the membrane potential of an unit crosses the firing threshold due to an excitatory stimulus then a neuronal firing occurs;
– after a neuronal firing a refractory period with fixed duration takes place, at the end of which the membrane potential is reset in proximity of the resting level according to a preassigned probability distribution;
– the effect of the neuronal firing of an unit is a jump for the membrane potential of all afferent neuronal units; these jumps have positive or negative amplitude depending on the excitatory or inhibitory nature of the firing neuron.

It should be stressed that two different streams of stimuli occur, both modeled by jumps for the membrane potential: one stream is due to external stimuli, the other is the effect of firings in the network's neuronal units.

Being very hard to obtain the analytical solution of the considered model we resort to Monte-Carlo simulations for the membrane potential dynamics. According to Schaefer *et al.* [2], the simulation procedure is based on a timeslot division of the simulation time. Our efforts are mainly devoted to investigate the input-output behaviour of the neuronal activity aiming to pinpoint the aspects related to the dependence of the firing times on the parameters that characterize the input process. Finally, the effects on firing times of different reset probability distributions are also elucidated.

Keywords: Neuronal models, stochastic simulation, firing times, neural networks.

References

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